

Transition magnetic moments between negative parity heavy baryons

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Abstract

The transition magnetic moments between negative parity, spin-1/2 heavy baryons are studied in framework of the light cone QCD sum rules. By constructing the sum rules for different Lorentz structures, the unwanted contributions coming from negative (positive) to positive (negative) parity transitions are removed. It is found that the magnetic moments between neutral negative parity heavy $\Xi_Q^{\prime 0}$ and Ξ_Q^0 baryons are very small. Magnetic moments of the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi_Q^{\prime \pm} \rightarrow \Xi_Q^{\pm}$ transitions are quite large and can be measured in further experiments.

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1 Introduction

Study of the electromagnetic properties of baryons constitutes very important source of information in exploring their internal structure, and can provide valuable insight in understanding the nonperturbative aspect of QCD. In recent years, study of the properties of heavy baryons has become a subject of growing interest due to the experimental observation of many heavy baryons. All ground state baryons containing single charm and bottom quarks, except Ω_b^* baryon, are observed and their masses are measured (for a review, see [1]). Moreover, a number of negative parity baryons have also been observed.

These exciting experimental results have stimulated the theoretical studies along these lines. The mass and magnetic moments of the heavy baryons can serve useful information about their inner structure. Experimentally the magnetic moments of all members of the octet $J^P = \frac{1}{2}^+$ baryons (except Σ^0 baryon), and two members of the decuplet $J^P = \frac{3}{2}^+$ baryons are measured [1].

The magnetic moments of the $J^P = \frac{1}{2}^+$ light and heavy baryons have extensively been calculated in numerous theoretical approaches. The approaches based on naive quark model in [2, 3], relativistic quark model [4], nonrelativistic quark model [5], chiral quark model [6], chiral perturbation theory [7], hypercentral model [8], soliton model [9] traditional [10] and light version of the QCD sum rules method (LCSR) [11–14] have been employed in studying the masses and magnetic moments of the heavy baryons. The magnetic moments of the negative parity heavy baryons has recently been considered within the framework of the LCSR method in [15].

In this work, we extend our analysis to determine the magnetic moments for the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions between the negative parity baryons.

In the following section we derive the light cone sum rules for the magnetic moments of the aforementioned transitions. Section 3 is devoted to the numerical analysis of the obtained sum rules for the transition magnetic moments. A comparison of our predictions with the results from other approaches is given also.

2 Theoretical framework

Following the philosophy of the QCD sum rules, the magnetic moments of the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions of the baryons with negative parity can be obtained in LCSR by matching two representations of the relevant correlation function written in terms of the hadronic and quark-gluon languages. For this purpose we use the correlation function

$$\Pi(p, q) = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_Q(x) \bar{\eta}_Q(0) \} | 0 \rangle_\gamma, \quad (1)$$

where γ is the external electromagnetic field, η_Q is the interpolating current of the heavy baryon with single heavy quark. This correlation function can be calculated at hadronic level by inserting a complete set of hadrons carrying the same quantum numbers of the correlation function, and isolating the contributions arising from the ground states which have poles in p^2 and $(p+q)^2$. The interpolating current can interact with both negative

and positive parity baryons, therefore it can be written in the following form,

$$\begin{aligned}
\Pi(p, q) = & \frac{\langle 0|\eta|B_2^{(+)}(p, s)\rangle}{p^2 - m_{B_2^{(+)}}^2} \langle B_2^{(+)}(p, s)\gamma(q)|B_1^{(+)}(p+q, s)\rangle \frac{\langle B_1^{(+)}(p+q, s)|\bar{\eta}(0)\rangle}{(p+q)^2 - m_{B_1^{(+)}}^2} \\
& + \frac{\langle 0|\eta|B_2^{(-)}(p, s)\rangle}{p^2 - m_{B_2^{(-)}}^2} \langle B_2^{(-)}(p, s)\gamma(q)|B_1^{(-)}(p+q, s)\rangle \frac{\langle B_1^{(-)}(p+q, s)|\bar{\eta}(0)\rangle}{(p+q)^2 - m_{B_1^{(-)}}^2} \\
& + \frac{\langle 0|\eta|B_2^{(+)}(p, s)\rangle}{p^2 - m_{B_2^{(+)}}^2} \langle B_2^{(+)}(p, s)\gamma(q)|B_1^{(-)}(p+q, s)\rangle \frac{\langle B_1^{(-)}(p+q, s)|\bar{\eta}(0)\rangle}{(p+q)^2 - m_{B_1^{(-)}}^2} \\
& + \frac{\langle 0|\eta|B_2^{(-)}(p, s)\rangle}{p^2 - m_{B_2^{(-)}}^2} \langle B_2^{(-)}(p, s)\gamma(q)|B_1^{(+)}(p+q, s)\rangle \frac{\langle B_1^{(+)}(p+q, s)|\bar{\eta}(0)\rangle}{(p+q)^2 - m_{B_1^{(+)}}^2} + \dots, \quad (2)
\end{aligned}$$

where $B^{(\pm)}$ and $m_{B^{(\pm)}}$ correspond to positive (negative) parity baryons and their masses, respectively; q is the photon momentum; and dots correspond to the higher states contributions.

The matrix elements in Eq. (2) are defined as,

$$\begin{aligned}
\langle 0|\eta|B^{(+)}(p)\rangle &= \lambda_{B^{(+)}} u^{(+)}(p), \\
\langle 0|\eta|B^{(-)}(p)\rangle &= \lambda_{B^{(-)}} \gamma_5 u^{(-)}(p), \\
\langle B_2^{(+)}(p)\gamma(q)|\eta|B_1^{(+)}(p+q)\rangle &= e\varepsilon^\mu \bar{u}^{(+)}(p) \left[\gamma_\mu f_1 - \frac{i\sigma_{\mu\nu} q^\nu}{m_{B_1^{(+)}} + m_{B_2^{(+)}}} f_2 \right] u^{(+)}(p+q) \\
&= e\varepsilon^\mu \bar{u}^{(+)}(p) \left[(f_1 + f_2) \gamma_\mu - \frac{(2p+q)_\mu}{m_{B_1^{(+)}} + m_{B_2^{(+)}}} f_2 \right] u^{(+)}(p+q), \\
\langle B_2^{(-)}(p)\gamma(q)|\eta|B_1^{(+)}(p+q)\rangle &= e\varepsilon^\mu \bar{u}^{(-)}(p) \left[\gamma_\mu f_1^T - \frac{i\sigma_{\mu\nu} q^\nu}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right] \gamma_5 u^{(+)}(p+q) \\
&= e\varepsilon^\mu \bar{u}^{(-)}(p) \left[\left(f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right) \gamma_\mu \right. \\
&\quad \left. - \frac{(2p+q)_\mu}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right] \gamma_5 u^{(+)}(p+q), \\
\langle B_2^{(-)}(p)\gamma(q)|\eta|B_1^{(-)}(p+q)\rangle &= e\varepsilon^\mu \bar{u}^{(-)}(p) \left[\gamma_\mu f_1^* - \frac{i\sigma_{\mu\nu} q^\nu}{m_{B_1^{(-)}} + m_{B_2^{(-)}}} f_2^* \right] u^{(-)}(p+q) \\
&= e\varepsilon^\mu \bar{u}^{(-)}(p) \left[(f_1^* + f_2^*) \gamma_\mu - \frac{(2p+q)_\mu}{m_{B_1^{(-)}} + m_{B_2^{(-)}}} f_2^* \right] u^{(-)}(p+q) \quad (3)
\end{aligned}$$

where ε^μ is the four-polarization vector.

Performing summation over spins of the heavy baryons, for the correlation function from the hadronic side we get,

$$\Pi(p, q) = A(\not{p}_2 + m_{B_2^{(+)}}) \not{q} (\not{p}_1 + m_{B_1^{(+)}})$$

$$\begin{aligned}
& + B(\not{p}_2 - m_{B_2^{(-)}}) \not{\epsilon}(\not{p}_1 - m_{B_1^{(-)}}) \\
& + C(\not{p}_2 - m_{B_2^{(-)}}) \not{\epsilon}\gamma_5(\not{p}_1 + m_{B_1^{(+)}}) \\
& + D(\not{p}_2 + m_{B_2^{(+)}}) \not{\epsilon}\gamma_5(\not{p}_1 - m_{B_1^{(-)}}) ,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
A &= \frac{\lambda_{B_1^{(+)}}\lambda_{B_2^{(+)}}(f_1 + f_2)}{(m_{B_1^{(+)}}^2 - p_1^2)(m_{B_2^{(+)}}^2 - p_2^2)} \\
B &= \frac{\lambda_{B_1^{(-)}}\lambda_{B_2^{(-)}}(f_1^* + f_2^*)}{(m_{B_1^{(-)}}^2 - p_1^2)(m_{B_2^{(-)}}^2 - p_2^2)} \\
C &= \frac{\lambda_{B_1^{(-)}}\lambda_{B_2^{(+)}}}{(m_{B_1^{(-)}}^2 - p_1^2)(m_{B_2^{(+)}}^2 - p_2^2)} \left[f_1^T + \frac{m_{B_1^{(-)}} - m_{B_2^{(+)}}}{m_{B_1^{(-)}} + m_{B_2^{(+)}}} f_2^T \right] \\
D &= \frac{\lambda_{B_1^{(+)}}\lambda_{B_2^{(-)}}}{(m_{B_1^{(+)}}^2 - p_1^2)(m_{B_2^{(-)}}^2 - p_2^2)} \left[f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right] ,
\end{aligned} \tag{5}$$

where $p_1 = p + q$ and $p_2 = p$.

Among the terms in Eq. (4)

$$f_1 + f_2 , \quad (f_1^* + f_2^*) , \quad f_1^T + \frac{m_{B_1^{(-)}} - m_{B_2^{(+)}}}{m_{B_1^{(-)}} + m_{B_2^{(+)}}} f_2^T , \quad f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T ,$$

that are proportional to γ_μ , the first two correspond to the magnetic moments of the positive to positive, negative to negative transitions, respectively; and the third and the fourth ones describe the transition magnetic moments between positive and negative parity baryons at $q^2 = 0$. Our aim in the present work is to calculate the transition magnetic moment between the negative parity baryons, and therefore we should find a way to remove the other three contributions.

In order to determine the transition magnetic moments between negative parity baryons four equations are needed, for which we choose the following four Lorentz structures, $(\varepsilon \cdot p)I$, $(\varepsilon \cdot p)\not{p}$, $\not{p}\not{\epsilon}$ and $\not{\epsilon}$. Solving finally these four coupled equations, we obtain the unknown coefficient B which describes the negative to negative parity transition.

It follows from Eq. (1) that interpolating currents are needed in order to calculate the correlation function in terms of quarks and gluons. Here, it should be remembered that hadrons containing single heavy quark belong to either sextet or anti-triplet representations of $SU(3)$. Sextet (anti-triplet) representation is symmetric (antisymmetric) with respect to the exchange of light quarks. In constructing the interpolating currents belonging to sextet and anti-triplet representations we will use this fact, whose explicit forms are given as (see [16]),

$$\begin{aligned}
\eta^{(s)} &= -\frac{1}{\sqrt{2}}\varepsilon^{abc} \left\{ (q_1^{aT} C Q^b) \gamma_5 q_2^c + t(q_1^{aT} C \gamma_5 Q^b) q_2^c + (q_2^{aT} C Q^b) \gamma_5 q_1^c + (q_2^{aT} C \gamma_5 Q^b) q_1^c \right\} , \\
\eta^{(a)} &= -\frac{1}{\sqrt{6}}\varepsilon^{abc} \left\{ 2(q_1^{aT} C q_2^b) \gamma_5 Q^c + 2t(q_1^{aT} C \gamma_5 q_2^b) Q^c + (q_1^{aT} C Q^b) \gamma_5 q_2^c \right. \\
&\quad \left. + t(q_1^{aT} C \gamma_5 Q^b) q_2^c - (q_2^{aT} C Q^b) \gamma_5 q_1^c - t(q_2^{aT} C \gamma_5 Q^b) q_1^c \right\} .
\end{aligned} \tag{6}$$

In this expression a, b, c are the color indices; C is the charge conjugation operator; and t is a free parameter (the choice $t = -1$ correspond to the Ioffe current). The light quark contents of the heavy Σ_Q , Ξ'_Q , Λ_Q and Ξ_Q baryons are presented in Table (1).

	$\Sigma_{c(b)}^{+(0)}$	$\Sigma_{c(b)}^{0(-)}$	$\Xi_{c(b)}^{n0(-)}$	$\Xi_{c(b)}^{n'+(0)}$	$\Lambda_{c(b)}^{+(0)}$	$\Xi_{c(b)}^{0(-)}$	$\Xi_{c(b)}^{+(0)}$
q_1	u	d	d	u	u	d	u
q_2	d	d	s	s	d	s	s

Table 1: Light quark contents of the heavy Σ_Q , Ξ'_Q , Λ_Q and Ξ_Q baryons.

Having the interpolating currents containing single heavy baryon at hand, the correlation function can easily be calculated. The correlation functions describing the sextet to anti-triplet transition magnetic moments in the light cone version of the sum rules contain the following contributions. The photon interacts with light or heavy quarks perturbatively. This contribution can be obtained by replacing one of the free quark operators by,

$$S^{free}(x) \rightarrow -\frac{1}{2} \int d^4y S^{free}(x-y) \gamma^\mu S^{free}(y) y^\nu \mathcal{F}_{\mu\nu} , \quad (7)$$

and the other two propagators are taken as the free quark operator. In Eq. (7) the Fock-Schwinger gauge, i.e., $A_\mu = \frac{1}{2} \mathcal{F}_{\mu\nu} y^\nu$ has been used. The other type of contribution can be calculated by replacing one of the propagators in the same manner as is given in Eq. (7), and replacing the other one (or both) by the “full” light quark operator.

Last type of contribution is the nonperturbative one, that can be obtained by replacing one of the light quark operators with,

$$S_{\mu\nu}^{ab} \rightarrow -\frac{1}{4} (q^a \Gamma_i q^b) (\Gamma_i)_{\mu\nu} , \quad (8)$$

where Γ_i are the full set of Dirac matrices; and the remaining two other propagators are taken as the full quark propagators. In calculating these contributions, the expressions of the light and heavy quark propagators in external field are needed. The light cone expansion of the propagator in external field is performed in [17], and it is found that the contributions of the three-particle $\bar{q}Gq$, and the four-particle $\bar{q}G^2q$, $\bar{q}q\bar{q}q$ nonlocal operators are small. Keeping this fact in mind, the expressions of the light and heavy quark propagators in external field are given by,

$$\begin{aligned} S_q(x) = & \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i\frac{m_q}{4} \not{x} \right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i\frac{m_q}{6} \not{x} \right) \\ & - ig_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - \frac{i}{4\pi^2 x^2} ux^\mu G_{\mu\nu}(ux) \gamma^\nu \right. \\ & \left. - i\frac{m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \frac{-x^2 \Lambda^2}{4} + 2\gamma_E \right) \right] , \end{aligned}$$

$$S_Q(x) = \frac{m_Q^2}{4\pi^2} \left\{ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{\not{x}}{(\sqrt{-x^2})^2} K_2(m_Q \sqrt{-x^2}) \right\} \\ - \frac{g_s}{16\pi^2} \int_0^1 du G_{\mu\nu}(ux) \left[(\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma^{\mu\nu} K_0(m_Q \sqrt{-x^2}) \right]. \quad (9)$$

Here Λ is the cut-off energy separating the perturbative and nonperturbative regions, K_i are the modified Bessel functions.

It should be noted here that, in the expression (8) which is used to calculate nonperturbative contributions, there appears the matrix elements of the nonlocal operators between vacuum and one photon states of the form $\langle \gamma(q) | \bar{q} \Gamma_i q | 0 \rangle$, in which all nonperturbative effects are encoded. These matrix elements are given in terms of the photon distribution amplitudes as [18],

$$\begin{aligned} \langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q} q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\ &- \frac{i}{2(qx)} e_q \langle \bar{q} q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\ \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q} q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\ &- \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\ &- \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\ &+ \left. \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\ &+ \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\ &- \left. \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \end{aligned}$$

$$\begin{aligned}
& - \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
& + \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \Big] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
& + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
& + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \Big\} , \tag{10}
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist-3, and $h_\gamma(u)$, \mathbb{A} , \mathcal{T}_i ($i = 1, 2, 3, 4$) are the twist-4 photon DAs, and χ is the magnetic susceptibility. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g) .$$

As has already been noted, in determination of the magnetic moment responsible for the negative to negative parity transition, four equations are needed, and for this purpose we choose the coefficients of the structures $(\varepsilon \cdot p)I$, $(\varepsilon \cdot p)\not{p}$, $\not{p}\not{\varepsilon}$ and $\not{\varepsilon}$. The sum rules for the negative to negative parity transition magnetic moments can be obtained by choosing the coefficients of the aforementioned Lorentz structures Π_i , and equate them to the corresponding coefficients in hadronic part. Solving then the linear equations for the coefficients describing the negative to negative transition magnetic moments, and performing Borel transformation over the variables $-p^2$ and $-(p+q)^2$ in order to suppress higher states and continuum contribution, we finally obtain the magnetic moment for the negative to negative parity baryon transitions as is given below,

$$\begin{aligned}
\mu = & \frac{e^{\frac{m_{B_1^{(-)}}^2/2M^2}} e^{\frac{m_{B_2^{(-)}}^2/2M^2}}}{2\lambda_{B_1^{(-)}}\lambda_{B_2^{(-)}} \left(m_{B_1^{(+)}} + m_{B_1^{(-)}} \right) \left(m_{B_2^{(+)}} + m_{B_2^{(-)}} \right)} \Big\{ \left(m_{B_1^{(+)}} + m_{B_2^{(-)}} \right) \left(\Pi_1^B - m_{B_2^{(+)}} \Pi_2^B \right) \right. \\
& \left. + 2m_{B_2^{(+)}} \Pi_3^B - 2\Pi_4^B \right\} . \tag{11}
\end{aligned}$$

In this expression we take $M_1^2 = M_2^2 = 2M^2$, since the masses of the Σ_Q , Λ_Q , Ξ'_Q and Ξ_Q baryons are very close to each other. The expressions of Π_i^B are presented in Appendix A.

It follows from Eq. (11) that in determination of the magnetic moments of the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions, the residues of the negative parity heavy baryons are necessary. These residues can be determined from the analysis of the two-point correlation function

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta_Q(x) \bar{\eta}_Q(0) \} | 0 \rangle ,$$

where η_Q is the interpolating current for the corresponding heavy baryon given by Eq. (6). This interpolating current interacts with both positive and negative parity heavy baryons. Saturating this correlation function with the ground states of positive and negative parity baryons we have,

$$\Pi(q^2) = \frac{|\lambda_{B^{(-)}}|^2 (\not{p} - m_{B^{(-)}})}{m_{B^{(-)}}^2 - p^2} + \frac{|\lambda_{B^{(+)}}|^2 (\not{p} + m_{B^{(+)}})}{m_{B^{(+)}}^2 - p^2} .$$

Eliminating the contributions coming from the positive parity baryons, the following sum rules for the residue and mass of the negative parity baryons are obtained,

$$|\lambda_{B^{(-)}}|^2 = \frac{1}{\pi} \frac{e^{m_{B^{(-)}}^2/M^2}}{m_{B^{(+)}} + m_{B^{(-)}}} \int ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)] ,$$

$$m_{B^{(-)}}^2 = \frac{\int_{m_Q^2}^{s_0} s ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)]}{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)]} .$$

Here Π_1^M and Π_2^M are the invariant functions corresponding to the structures \not{p} and I , respectively. The expressions of Π_1^M and Π_2^M for the Σ_Q^0 baryon are presented in Appendix B.

3 Numerical analysis

In this section we present our numerical results for the magnetic moments of the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions for negative parity baryons derived from the LCSR. In this numerical analysis the values of the relevant input parameters entering to the LCSR are needed. The main nonperturbative input of LCSR is the DAs which are all calculated in [18], and for completeness we present their expressions in Appendix C. The other input parameters needed in the numerical analysis are, quark condensate $\langle \bar{q}q \rangle$, m_0^2 , magnetic susceptibility χ of quarks, etc. In further numerical calculations we use $[\langle \bar{u}u \rangle = \langle \bar{d}d \rangle]_{\mu=1 \text{ GeV}} = -(0.243)^3 \text{ GeV}^3$ [19], $\langle \bar{s}s \rangle|_{\mu=1 \text{ GeV}} = 0.8 \langle \bar{u}u \rangle|_{\mu=1 \text{ GeV}}$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [20]. The magnetic susceptibility was determined within the QCD sum rules in [21–23]).

Having all necessary ingredients at hand, we are now ready to perform the numerical analysis for the transition magnetic moments of the negative parity baryons. Sum rules contain also three auxiliary parameters in the interpolating current other than those input parameters given above: Borel mass parameter M^2 , continuum threshold s_0 , and the arbitrary parameter t . We demand that the magnetic moment should be independent on these auxiliary parameters. Therefore we shall look for the “working regions” of these parameters, where magnetic moments exhibit good stability with respect to their variations in respective domains. It should be remembered that the continuum threshold s_0 is not arbitrary but related to the first excited states. The difference $\sqrt{s_0} - m_{\text{ground}}$ is the energy needed to transfer the baryon to its first excited state. Usually this difference varies in the range $0.3 \text{ GeV} \leq \sqrt{s_0} - m_{\text{ground}} \leq 0.8 \text{ GeV}$, and in our analysis we choose the average value $\sqrt{s_0} - m_{\text{ground}} = 0.5 \text{ GeV}$.

Having determined the value of s_0 , next we try to find the “working regions” of the Borel parameter M^2 . The upper bound of M^2 is obtained by demanding that contributions of higher states and continuum constitute about 40% of the perturbative part. The lower bound is determined from the condition that higher twist contributions are less than the leading twist contributions. Our analysis shows that the working regions of M^2 where both conditions are satisfied are

$$2.5 \text{ GeV}^2 \leq M^2 \leq 4.0 \text{ GeV}^2, \text{ for } \Sigma_c, \Xi'_c, \Lambda_c, \Xi_c ,$$

$$4.5 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2, \text{ for } \Sigma_b, \Xi'_b, \Lambda_b, \Xi_b .$$

As an example, in Figs. (1) and (2) we present the dependence of $\mu_{\Sigma_b^0 \rightarrow \Lambda_b^0}$ on M^2 at several fixed values of the arbitrary parameter t , at $s_0 = 40.0 \text{ GeV}^2$ and $s_0 = 42.5 \text{ GeV}^2$. We observe from these figures that transition magnetic moment exhibits good stability when M^2 varies in the region $3.0 \text{ GeV}^2 \leq M^2 \leq 4.0 \text{ GeV}^2$. In Figs. (3) and (4) we present the dependence of $\mu_{\Sigma_b^0 \rightarrow \Lambda_b^0}$ on $\cos \theta$ (where $t = \tan \theta$ at two fixed values of M^2 and at $s_0 = 40.0 \text{ GeV}^2$ and $s_0 = 42.5 \text{ GeV}^2$, respectively. We see from these figures that, when $\cos \theta$ varies in the domain $-1.0 \leq \cos \theta \leq -0.7$, the magnetic moment demonstrates good stability with respect to the variation in $\cos \theta$. Our final result for the transition magnetic moment is $\mu_{\Sigma_b^0 \rightarrow \Lambda_b^0} = (-0.3 \pm 0.05)\mu_N$.

The analysis of the sum rules for the other transition magnetic moments are also calculated, whose values can be summarized as,

$$\begin{aligned}\mu_{\Sigma_c^+ \rightarrow \Lambda_c^+} &= (0.25 \pm 0.05)\mu_N, \\ \mu_{\Xi_c'^0 \rightarrow \Xi_c^0} &= (0.08 \pm 0.01)\mu_N, \\ \mu_{\Xi_c'^+ \rightarrow \Xi_c^+} &= (0.20 \pm 0.05)\mu_N, \\ \mu_{\Xi_b'^0 \rightarrow \Xi_b^0} &= (-0.008 \pm 0.001)\mu_N, \\ \mu_{\Xi_b'^- \rightarrow \Xi_b^-} &= (0.10 \pm 0.01)\mu_N,\end{aligned}$$

where upper signs correspond to the electric charge of the corresponding negative parity baryons. It can easily be seen from these results that the transition magnetic moments between the neutral Ξ' and Ξ baryons are very close to zero. The magnetic moments for the $\Sigma_c^+ \rightarrow \Lambda_c^+$ and $\Xi_b'^+ \rightarrow \Xi_b^+$ transitions are very close to each other which follows from $SU(3)$ symmetry arguments.

In conclusion, the transition magnetic moments of the negative parity, spin-1/2 heavy baryons are estimated within the QCD sum rules. The contributions coming from the positive to positive, as well as positive to negative parity transitions are eliminated by constructing various sum rules. It is obtained that the magnetic moments between neutral, negative parity heavy Ξ_Q^0 and Ξ_Q^0 baryons are very small. Moreover, it is found that the magnetic moments for the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi_Q'^{\pm} \rightarrow \Xi_Q^{\pm}$ transitions of the negative parity heavy baryons are quite large and can be measured in future experiments.

Appendix A

In this Appendix we present the expressions of the invariant functions Π_i^B appearing in the sum rules for the magnetic moment of $\Xi_b^0 \rightarrow \Xi_b^0$ transition. Here in this appendix, and in appendix B the masses of the light quarks are neglected.

1) Coefficient of the $(\varepsilon \cdot p)I$ structure

$$\begin{aligned}
\Pi_1^B = & -\frac{1}{32\sqrt{3}\pi^2}(-1+t)m_b^2M^4\left\{4(2+t)m_b^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\mathcal{I}_3\right. \\
& + e_b(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\left[(7+3t)\mathcal{I}_2 - 2(3+t)m_b^2\mathcal{I}_3\right]\Big\} \\
& + \frac{\sqrt{3}}{8\pi^2}(-1+t)m_b^4M^4(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\mathcal{I}_3\tilde{j}(h_\gamma) \\
& + \frac{1}{16\sqrt{3}\pi^2}(-1+t)(3+t)(e_s - e_u)f_{3\gamma}m_b^3M^4(\mathcal{I}_2 - m_b^2\mathcal{I}_3)\psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{768\sqrt{3}\pi^2}(-1+t)M^2\left\{12m_0^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[4+t\left(2+m_b^2e^{m_b^2/M^2}\mathcal{I}_2\right)\right]\right. \\
& + e_b(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\left[24(7+3t)m_b^2e^{m_b^2/M^2}(-\mathcal{I}_1 + m_b^2\mathcal{I}_2) + m_0^2\left(7(1+t) + (29+17t)m_b^2e^{m_b^2/M^2}\mathcal{I}_2\right)\right]\Big\} \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}m_b\pi^2}(-1+t)f_{3\gamma}M^2\left\{e_u\left[-96(1+t)m_b\pi^2\langle\bar{s}s\rangle - (3+t)\langle g_s^2G^2\rangle\left(-1+3m_b^2e^{m_b^2/M^2}\mathcal{I}_2\right)\right]\right. \\
& + e_s\left[96(1+t)m_b\pi^2\langle\bar{u}u\rangle + (3+t)\langle g_s^2G^2\rangle\left(-1+3m_b^2e^{m_b^2/M^2}\mathcal{I}_2\right)\right]\Big\}\psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{48\sqrt{3}M^2}(-1+t^2)f_{3\gamma}m_0^2m_b^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^4\pi^2}(-1+t)\langle g_s^2G^2\rangle m_b^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[-3(2+t)m_0^2 + 8(1+t)f_{3\gamma}\pi^2\psi^v(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{1728\sqrt{3}M^6}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\psi^v(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{3456\sqrt{3}M^8}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^4(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2}(-1+t)\left[4(2+t)\langle g_s^2G^2\rangle(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) + 3(29+17t)e_bm_0^2m_b^2e^{m_b^2/M^2}(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\mathcal{I}_1\right] \\
& - \frac{e^{-m_b^2/M^2}}{192\sqrt{3}\pi^2}(-1+t)\langle g_s^2G^2\rangle(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\tilde{j}(h_\gamma) \\
& - \frac{e^{-m_b^2/M^2}}{288\sqrt{3}}(-1+t^2)f_{3\gamma}m_0^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\psi^v(u_0) .
\end{aligned}$$

2) Coefficient of the $(\varepsilon \cdot p) \not{p}$ structure

$$\begin{aligned}
\Pi_2^B = & \frac{1}{8\sqrt{3}\pi^2}(-2+t+t^2)m_b^3M^2 \left[(e_b+e_u)\langle \bar{s}s \rangle - (e_b+e_s)\langle \bar{u}u \rangle \right] (\mathcal{I}_2 - m_b^2\mathcal{I}_3) \\
& - \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}m_bM^2\pi^2}(-2+t+t^2)\langle g_s^2G^2 \rangle m_0^2 (e_u\langle \bar{s}s \rangle - e_s\langle \bar{u}u \rangle) \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}M^4\pi^2}(-2+t+t^2)\langle g_s^2G^2 \rangle m_0^2 m_b (e_u\langle \bar{s}s \rangle - e_s\langle \bar{u}u \rangle) \\
& - \frac{e^{-m_b^2/M^2}}{576\sqrt{3}m_b\pi^2}(-1+t)(2+t)\langle g_s^2G^2 \rangle (e_u\langle \bar{s}s \rangle - e_s\langle \bar{u}u \rangle) \\
& + \frac{\sqrt{3}}{64\pi^2}(-1+t^2)m_0^2m_b (e_u\langle \bar{s}s \rangle - e_s\langle \bar{u}u \rangle) \mathcal{I}_1 \\
& + \frac{1}{192\sqrt{3}\pi^2}(-1+t)m_b \left\{ \left[(2+t)e_u\langle g_s^2G^2 \rangle - 3(3+2t)e_b m_0^2 m_b^2 - 3(7+5t)e_u m_0^2 m_b^2 \right] \langle \bar{s}s \rangle \right. \\
& \left. + \left[-(2+t)e_s\langle g_s^2G^2 \rangle + 3(3+2t)e_b m_0^2 m_b^2 + 3(7+5t)e_s m_0^2 m_b^2 \right] \langle \bar{u}u \rangle \right\} \mathcal{I}_2 .
\end{aligned}$$

3) Coefficient of the $\not{p}\not{p}$ structure

$$\begin{aligned}
\Pi_3^B = & \frac{1}{256\sqrt{3}\pi^4}(-1+t)m_b^3M^6 \left[-3(3+t)(e_s-e_u) (\mathcal{I}_2 - 2m_b^2\mathcal{I}_3 + m_b^4\mathcal{I}_4) \right. \\
& \left. + 8(-1+t)m_b\pi^2 (e_s\langle \bar{s}s \rangle - e_u\langle \bar{u}u \rangle) \chi (\mathcal{I}_3 - m_b^2\mathcal{I}_4) \varphi'_\gamma(u_0) \right] \\
& - \frac{1}{768\sqrt{3}\pi^4}(-1+t)(3+t)m_b^3M^4 \left[\langle g_s^2G^2 \rangle (e_u-e_s) + 24(e_b-e_u)m_b\pi^2\langle \bar{s}s \rangle \right. \\
& \left. + 24(-e_b+e_s)m_b\pi^2\langle \bar{u}u \rangle \right] \mathcal{I}_3 \\
& + \frac{1}{1024\sqrt{3}\pi^4}(e_s-e_u)m_bM^4 \left\{ -(-1+t)(3+t)\langle g_s^2G^2 \rangle \mathcal{I}_2 + 16f_{3\gamma}m_b^2\pi^2 (-\mathcal{I}_2 + m_b^2\mathcal{I}_3) \right. \\
& \left. \times \left[2(-1+t)(3+t)\psi^v(u_0) - (-1+t^2)\psi^{a'}(u_0) \right] \right\} \\
& + \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b^2M^4 (e_s\langle \bar{s}s \rangle - e_u\langle \bar{u}u \rangle) \mathcal{I}_2 \left\{ (5+t)i_1(\mathcal{S}, 1) + (1+5t)i_1(\tilde{\mathcal{S}}, 1) \right. \\
& + 2i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) - 5i_1(\mathcal{T}_4, 1) - 6i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) \\
& - t \left[2i_1(\mathcal{T}_1, 1) - 5i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) + i_1(\mathcal{T}_4, 1) + 2i_1(\mathcal{S}, v) + 6i_1(\tilde{\mathcal{S}}, v) \right. \\
& \left. + 4i_1(\mathcal{T}_2, v) - 4i_1(\mathcal{T}_3, v) \right] - 4i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) \left\{ \right. \\
& \left. - \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b^4M^4 (e_s\langle \bar{s}s \rangle - e_u\langle \bar{u}u \rangle) \mathcal{I}_3 \left\{ 4(2+t)i_1(\mathcal{S}, 1) + (4+8t)i_1(\tilde{\mathcal{S}}, 1) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -4 \left[(-1+t)i_1(\mathcal{T}_1, 1) - i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_4, 1) + 3i_1(\mathcal{S}, v) + i_1(\tilde{\mathcal{S}}, v) + i_1(\mathcal{T}_2, v) \right. \\
& + t \left(-2i_1(\mathcal{T}_2, 1) + i_1(\mathcal{T}_4, 1) + i_1(\mathcal{S}, v) + 3i_1(\tilde{\mathcal{S}}, v) + i_1(\mathcal{T}_2, v) \right) \Big] \\
& + 4(1+t)i_1(\mathcal{T}_4, v) + 8(2+t)\tilde{j}(h_\gamma) + (-1+t)\mathbb{A}'(u_0) \Big\} \\
& + \frac{e^{-m_b^2/M^2}}{768\sqrt{3}\pi^2} (-1+t)M^2 \left\{ m_0^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[-6(3+t) + (7+t)m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right] \right. \\
& + e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \left[(11+5t)m_0^2 - 24(3+t)m_b^2 e^{m_b^2/M^2} (-\mathcal{I}_1 + m_b^2 \mathcal{I}_2) \right] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}m_b\pi^2} (-1+t)(3+t)f_{3\gamma}M^2 \left[- (e_s - e_u) \langle g_s^2 G^2 \rangle - 96m_b\pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& + 3(e_s - e_u) \langle g_s^2 G^2 \rangle m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \Big] \psi^v(u_0) \\
& - \frac{1}{2304\sqrt{3}\pi^2} (-1+t)^2 \langle g_s^2 G^2 \rangle m_b^2 M^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi \mathcal{I}_2 \varphi'_\gamma(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}m_b\pi^2} f_{3\gamma}M^2 \left[96t(-1+t)m_b\pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& + (-1+t^2)(e_s - e_u) \langle g_s^2 G^2 \rangle \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \Big] \psi^{a'}(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{192\sqrt{3}M^2} (-1+t)f_{3\gamma}m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{27648\sqrt{3}M^4\pi^2} (-1+t) \langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ 3(3+t)m_0^2 \right. \\
& - 8f_{3\gamma}\pi^2 \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^6} (-1+t)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^8} (-1+t)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}\pi^2} (-1+t) \langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ 3(1+t)i_1(\mathcal{S}, 1) + 3(1+t)i_1(\tilde{\mathcal{S}}, 1) \right. \\
& + 2i_1(\mathcal{T}_1, 1) + 3i_1(\mathcal{T}_2, 1) - 2i_1(\mathcal{T}_3, 1) - 3i_1(\mathcal{T}_4, 1) - 6i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) - 4i_1(\mathcal{T}_2, v) \\
& + 4i_1(\mathcal{T}_3, v) + 16\tilde{j}(h_\gamma) - \mathbb{A}'(u_0) + t \left[-2i_1(\mathcal{T}_1, 1) + 3i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) - 3i_1(\mathcal{T}_4, 1) \right. \\
& - 2i_1(\mathcal{S}, v) - 6i_1(\tilde{\mathcal{S}}, v) - 4i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) + 8\tilde{j}(h_\gamma) + \mathbb{A}'(u_0) \Big] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2} (-1+t) \left\{ (3+t) \langle g_s^2 G^2 \rangle (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) + 3(11+5t)e_b m_0^2 m_b^2 e^{m_b^2/M^2} (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \mathcal{I}_1 \right. \\
& + 2f_{3\gamma}m_0^2 \pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(11+5t)\psi^v(u_0) + (2+5t)\psi^{a'}(u_0) \right] \Big\} .
\end{aligned}$$

4) Coefficient of the \not{e} structure

$$\begin{aligned}
\Pi_4^B = & \frac{\sqrt{3}}{32\pi^4}(1+t+t^2)(e_s - e_u)m_b^4 M^8 (-\mathcal{I}_3 + m_b^2 \mathcal{I}_4) \\
& + \frac{1}{128\sqrt{3}\pi^2}(e_s - e_u)f_{3\gamma}m_b^2 M^6 [-3(1+t)^2 \mathcal{I}_2 + 4(1+t+t^2)m_b^2 \mathcal{I}_3] i_2(\mathcal{A}, v) \\
& + \frac{1}{128\sqrt{3}\pi^2}(e_s - e_u)f_{3\gamma}m_b^2 M^6 [-3(1+t)^2 \mathcal{I}_2 + 2(1+4t+t^2)m_b^2 \mathcal{I}_3] i_2(\mathcal{V}, v) \\
& - \frac{1}{32\sqrt{3}\pi^2}(1+t+t^2)(e_s - e_u)f_{3\gamma}m_b^4 M^6 \mathcal{I}_3 [4\psi^v(u_0) - \psi^{a'}(u_0)] \\
& + \frac{\sqrt{3}}{64\pi^2}(-1+t^2)m_b^3 M^6 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi (\mathcal{I}_2 - m_b^2 \mathcal{I}_3) \varphi'_\gamma(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{1536\sqrt{3}m_b\pi^2}(-1+t^2)\langle g_s^2 G^2 \rangle M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \varphi'_\gamma(u_0) \\
& - \frac{1}{16\sqrt{3}\pi^2}(-2+t+t^2)m_b^5 M^4 [(e_b + e_u) \langle \bar{s}s \rangle - (e_b + e_s) \langle \bar{u}u \rangle] \mathcal{I}_3 \\
& - \frac{\sqrt{3}}{128\pi^2}(-1+t^2)m_b M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathcal{I}_1 [i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_3, 1)] \\
& - \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathcal{I}_1 \left\{ (5+t)i_1(\mathcal{S}, 1) \right. \\
& - (1+5t) \left[i_1(\tilde{\mathcal{S}}, 1) + i_1(\mathcal{T}_2, 1) \right] - (5+t)i_1(\mathcal{T}_4, 1) \left. \right\} \\
& - \frac{1}{768\sqrt{3}\pi^4}m_b^2 M^4 \mathcal{I}_2 \left\{ -(1+t+t^2)e_s \langle g_s^2 G^2 \rangle + (1+t+t^2)e_u \langle g_s^2 G^2 \rangle \right. \\
& - 48(-2+t+t^2)e_b m_b \pi^2 (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) + 3(-1+t)m_b \pi^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \\
& \times \left[2(-1+7t) \left(i_1(\tilde{\mathcal{S}}, 1) + i_1(\mathcal{T}_2, 1) \right) + 2(-7+t) (i_1(\mathcal{S}, 1) - i_1(\mathcal{T}_4, 1)) \right. \\
& + 8(2+t)i_1(\mathcal{S}, v) - 4(-1+t)i_1(\tilde{\mathcal{S}}, v) + 8ti_1(\mathcal{T}_4, v) + 4(3+t) \left(i_1(\mathcal{T}_2, v) - 2\tilde{j}(h_\gamma) \right) \\
& + 3(1+t) (-4(i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_3, v)) + \mathbb{A}'(u_0)) \left. \right] \left. \right\} \\
& - \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}\pi^2}(-1+t)^2(e_s - e_u)f_{3\gamma}\langle g_s^2 G^2 \rangle M^2 [i_2(\mathcal{A}, v) - i_2(\mathcal{V}, v)] \\
& - \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}m_b\pi^2}(-1+t)\langle g_s^2 G^2 \rangle M^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ 4(-1+t)i_1(\mathcal{S}, 1) \right. \\
& + 2 \left[2(-1+t)i_1(\tilde{\mathcal{S}}, 1) - 3(1+t)i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) \right. \\
& + 2 \left(i_1(\mathcal{T}_4, 1) + 4i_1(\mathcal{S}, v) + i_1(\tilde{\mathcal{S}}, v) + 3i_1(\mathcal{T}_2, v) - 3i_1(\mathcal{T}_3, v) - 6\tilde{j}(h_\gamma) \right) \\
& + t \left(2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) - 2i_1(\mathcal{T}_4, 1) + 4i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) + 2i_1(\mathcal{T}_2, v) \right. \\
& - 6i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) - 4\tilde{j}(h_\gamma) \left. \right) \left. \right] + 3(1+t)\mathbb{A}'(u_0) \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{128\sqrt{3}\pi^2}(-1+t)(7+5t)m_0^2m_b^3M^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\mathcal{I}_2 \\
& - \frac{e^{-m_b^2/M^2}}{384\sqrt{3}m_b\pi^2}(-2+t+t^2)M^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[\langle g_s^2G^2\rangle\left(1 - e^{m_b^2/M^2}m_b^2\mathcal{I}_2\right)\right. \\
& \left. - 6m_0^2m_b^2 + 32f_{3\gamma}m_b^2\pi^2\psi^v(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2}(1+t+t^2)(e_s - e_u)f_{3\gamma}\langle g_s^2G^2\rangle M^2[4\psi^v(u_0) - \psi^{a'}(u_0)] \\
& + \frac{e^{-m_b^2/M^2}}{48\sqrt{3}}(1+t-2t^2)e_sf_{3\gamma}m_bM^2\langle\bar{u}u\rangle\psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{384\sqrt{3}\pi^2}(-1+t)m_bM^2\left[-e^{m_b^2/M^2}3(3+2t)e_bm_0^2m_b^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\mathcal{I}_2\right. \\
& \left. + 8(1+2t)e_sf_{3\gamma}\pi^2\langle\bar{s}s\rangle\psi^{a'}(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}M^2\pi^2}(-1+t)m_b(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{(2+t)\langle g_s^2G^2\rangle m_0^2\right. \\
& \left.+ f_{3\gamma}(\langle g_s^2G^2\rangle - 6m_0^2m_b^2)\pi^2\left[-4(2+t)\psi^v(u_0) + (1+2t)\psi^{a'}(u_0)\right]\right\} \\
& - \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^4\pi^2}(-1+t)\langle g_s^2G^2\rangle m_b(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{3(2+t)m_0^2m_b^2\right. \\
& \left.+ 2f_{3\gamma}(3m_0^2 - 2m_b^2)\pi^2\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right]\right\} \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}M^6}(-1+t)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^3(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right] \\
& - \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^8}(-1+t)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^5(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{18432\sqrt{3}\pi^2}(-1+t)\langle g_s^2G^2\rangle m_b(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\left\{4(-1+t)i_1(\mathcal{S}, 1)\right. \\
& + 2\left[2(-1+t)i_1(\tilde{\mathcal{S}}, 1) - 3(1+t)i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) + 2i_1(\mathcal{T}_4, 1) + 8i_1(\mathcal{S}, v)\right. \\
& + 2i_1(\tilde{\mathcal{S}}, v) + 6i_1(\mathcal{T}_2, v) - 6i_1(\mathcal{T}_3, v) - 12\tilde{j}(h_\gamma) + t\left(2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) - 2i_1(\mathcal{T}_4, 1)\right. \\
& \left. + 4i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) + 2i_1(\mathcal{T}_2, v) - 6i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) - 4\tilde{j}(h_\gamma)\right)\left. + 3(1+t)\mathbb{A}'(u_0)\right\} \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}m_b\pi^2}(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{-(-2+t+t^2)\langle g_s^2G^2\rangle(m_0^2 - 2m_b^2)\right. \\
& \left.+ 18(-1+t^2)f_{3\gamma}m_0^2m_b^2\pi^2\left[-4\psi^v(u_0) + \psi^{a'}(u_0)\right]\right\}.
\end{aligned}$$

The functions i_n ($n = 1, 2$), and $\tilde{j}_1(f(u))$ are defined as:

$$i_1(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0) ,$$

$$\begin{aligned}
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta''(k - u_0) , \\
\tilde{j}(f(u)) &= \int_{u_0}^1 du f(u) , \\
\mathcal{I}_n &= \int_{m_b^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n} ,
\end{aligned}$$

where

$$k = \alpha_q + \alpha_g \bar{v} , \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

Appendix B

Expressions of the invariant amplitudes Π_1^M and Π_2^M entering into the mass sum rule for the negative parity heavy Ξ_b^0 baryon.

1) Coefficient of the \not{p} structure

$$\begin{aligned}
\Pi_1^M = & \frac{3}{256\pi^4} \left\{ -m_b^4 M^6 [5 + t(2 + 5t)] [m_b^4 \mathcal{I}_5 - 2m_b^2 \mathcal{I}_4 + \mathcal{I}_3] \right\} \\
& + \frac{1}{192\pi^4} m_b^4 M^2 [\langle g_s^2 G^2 \rangle (1 + t + t^2) - 18m_b \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_3 \\
& + \frac{1}{3072\pi^4} m_b^2 M^2 [-\langle g_s^2 G^2 \rangle (13 + 10t + 13t^2) + 288m_b \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_2 \\
& + \frac{e^{-m_b^2/M^2}}{73728m_b M^2 \pi^4} \left\{ -\langle g_s^2 G^2 \rangle^2 m_b (1 + t)^2 + 768m_b m_0^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (-1 + t)^2 \right. \\
& \left. - 56\langle g_s^2 G^2 \rangle m_0^2 \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \right\} \\
& + \frac{1}{768M^2 \pi^2} \langle g_s^2 G^2 \rangle m_b (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \mathcal{E}_1 \\
& + \frac{e^{-m_b^2/M^2}}{18432M^4 \pi^2} m_b m_0^2 [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) + 384m_b \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} m_b^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{1728M^8} m_b^2 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^4 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{768m_b \pi^2} [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) + 32m_b \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
& + \frac{1}{256\pi^2} m_b (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) [(\langle g_s^2 G^2 \rangle - 13m_b^2 m_0^2) \mathcal{I}_2 + 6m_0^2 \mathcal{I}_1] ,
\end{aligned}$$

2) Coefficient of the I structure

$$\begin{aligned}
\Pi_2^M = & -\frac{3}{256\pi^4} \left\{ -m_b^3 M^6 (-1 + t)^2 [m_b^4 \mathcal{I}_4 - 2m_b^2 \mathcal{I}_3 + \mathcal{I}_2] \right\} \\
& + \frac{1}{3072\pi^4} m_b M^4 \left\{ 4m_b^2 [\langle g_s^2 G^2 \rangle (-1 + t)^2 + 72m_b (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1 + t^2)] \mathcal{I}_3 - 3\langle g_s^2 G^2 \rangle (-1 + t)^2 \mathcal{I}_2 \right\} \\
& - \frac{7e^{-m_b^2/M^2}}{256\pi^2} m_0^2 M^2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) \\
& + \frac{1}{1024\pi^4} m_b M^2 \left\{ m_b [3m_b \langle g_s^2 G^2 \rangle (-1 + t)^2 + 4m_0^2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1 + t^2)] \mathcal{I}_2 - 2\langle g_s^2 G^2 \rangle (-1 + t)^2 \mathcal{I}_1 \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^{-m_b^2/M^2}}{73728M^2\pi^4} m_b \left[\langle g_s^2 G^2 \rangle^2 (-1+t)^2 + 1536m_0^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (3+2t+3t^2) \right] \\
& + \frac{e^{-m_b^2/M^2}}{18432M^4\pi^2} m_b \left[-11m_b m_0^2 \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1+t^2) \right. \\
& - 32 (\langle g_s^2 G^2 \rangle - 12m_0^2 m_b^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \left. \right] \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} m_b (m_b^2 - 3m_0^2) \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{576M^8} m_b^3 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^5 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{36864m_b\pi^4} \left[\langle g_s^2 G^2 \rangle^2 (-1+t)^2 - 1536m_b^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (5+2t+5t^2) \right. \\
& + 96m_b \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1+t^2) \left. \right] .
\end{aligned}$$

Expressions of the invariant amplitudes Π_1^M and Π_2^M entering into the mass sum rule for the negative parity heavy Ξ_b^0 baryon.

3) Coefficient of the \not{p} structure

$$\begin{aligned}
\Pi_1^M &= -\frac{1}{256\pi^4} (-3m_b^4 M^6 (5+2t+5t^2) (\mathcal{I}_3 - 2m_b^2 \mathcal{I}_4 + m_b^4 \mathcal{I}_5) \\
& + \frac{1}{3072\pi^4} m_b^2 M^2 \left[3\langle g_s^2 G^2 \rangle (1+t)^2 \mathcal{I}_2 - 16\langle g_s^2 G^2 \rangle m_b^2 (1+t+t^2) \mathcal{I}_3 \right. \\
& - 32m_b \pi^2 (-1+t)(1+5t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) (-\mathcal{I}_2 + m_b^2 \mathcal{I}_3) \left. \right] \\
& + \frac{e^{-m_b^2/M^2}}{221184m_b M^2 \pi^4} \langle g_s^2 G^2 \rangle^2 m_b (13+10t+13t^2) + 768m_0^2 m_b \pi^4 (-1+t)(25+23t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - 8\langle g_s^2 G^2 \rangle \pi^2 (-1+t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \left[m_0^2 (1+5t) + 12m_b^2 e^{m_b^2/M^2} (5+t) \mathcal{I}_1 \right] \\
& + \frac{e^{-m_b^2/M^2}}{55296M^4\pi^2} m_0^2 m_b (-1+t) \left[384m_b \pi^2 (13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle + \langle g_s^2 G^2 \rangle (31+11t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \right] \\
& + \frac{e^{-m_b^2/M^2}}{5184M^6} \langle g_s^2 G^2 \rangle m_b^2 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{5184M^8} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{10368M^{10}} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{6912m_b\pi^2} (-1+t) \left\{ \langle g_s^2 G^2 \rangle (1+5t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right.
\end{aligned}$$

$$- 3m_b \left[32\pi^2(13 + 11t)\langle\bar{s}s\rangle\langle\bar{u}u\rangle + 3m_0^2m_b e^{m_b^2/M^2} (\langle\bar{s}s\rangle + \langle\bar{u}u\rangle) [-6(1+t)\mathcal{I}_1 + m_b^2(7+11t)\mathcal{I}_2] \right] \} .$$

4) Coefficient of the I structure

$$\begin{aligned} \Pi_2^M = & -\frac{1}{256\pi^4}m_b^3M^6(-1+t)(13+11t) (\mathcal{I}_2 - 2m_b^2\mathcal{I}_3 + m_b^4\mathcal{I}_4) \\ & - \frac{1}{9216\pi^4}m_bM^4(-1+t) \left[-96m_b^3\pi^2(1+5t)(\langle\bar{s}s\rangle + \langle\bar{u}u\rangle)\mathcal{I}_3 + \langle g_s^2G^2\rangle(13+11t) (3\mathcal{I}_2 - 4m_b^2\mathcal{I}_3) \right] \\ & - \frac{e^{-m_b^2/M^2}}{3072\pi^4}M^2(-1+t) \left\{ 4m_0^2\pi^2 (\langle\bar{s}s\rangle + \langle\bar{u}u\rangle) \left[1+5t + m_b^2e^{m_b^2/M^2}(5+t)\mathcal{I}_2 \right] \right. \\ & + \langle g_s^2G^2\rangle m_b e^{m_b^2/M^2} \left[-2(-1+t)\mathcal{I}_1 + 3m_b^2(3+5t)\mathcal{I}_2 \right] \} \\ & + \frac{e^{-m_b^2/M^2}}{221184M^2\pi^4}m_b(-1+t) \left[\langle g_s^2G^2\rangle^2(11+13t) - 1536m_0^2\pi^4(-1+t)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \right] \\ & + \frac{e^{-m_b^2/M^2}}{55296M^4\pi^2}m_b \left\{ 1152m_0^2m_b^2\pi^2(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \right. \\ & - \langle g_s^2G^2\rangle \left[96\pi^2(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle + m_0^2m_b(-1+t)(29+t) (\langle\bar{s}s\rangle + \langle\bar{u}u\rangle) \right] \} \\ & + \frac{e^{-m_b^2/M^2}}{1728M^6}\langle g_s^2G^2\rangle m_b(-3m_0^2 + m_b^2)(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \\ & + \frac{e^{-m_b^2/M^2}}{576M^8}\langle g_s^2G^2\rangle m_0^2m_b^3(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \\ & - \frac{e^{-m_b^2/M^2}}{3456M^{10}}\langle g_s^2G^2\rangle m_0^2m_b^5(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \\ & + \frac{e^{-m_b^2/M^2}}{110592m_b\pi^4} \left[\langle g_s^2G^2\rangle^2(11+2t-13t^2) - 4608m_b^2\pi^4(5+2t+5t^2)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \right. \\ & \left. - 32\langle g_s^2G^2\rangle m_b\pi^2(-7+t)(-1+t) (\langle\bar{s}s\rangle + \langle\bar{u}u\rangle) \right] . \end{aligned}$$

where

$$\mathcal{I}_n = \int_{m_b^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n} .$$

Appendix C: Photon distribution amplitudes

Explicit forms of the photon DAs [18].

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u} \left[1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right] , \\
\psi^v(u) &= 3[3(2u - 1)^2 - 1] + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)[3 - 30(2u - 1)^2 + 35(2u - 1)^4] , \\
\psi^a(u) &= [1 - (2u - 1)^2][5(2u - 1)^2 - 1] \frac{5}{2} \left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A \right) , \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2 \left[1 + w_\gamma^A \frac{1}{2}(7\alpha_g - 3) \right] , \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2 , \\
h_\gamma(u) &= -10(1 + 2\kappa^+)C_2^{\frac{1}{2}}(u - \bar{u}) , \\
\mathbb{A}(u) &= 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) \\
&\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})] , \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g , \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)] , \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g , \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)] , \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\} , \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\} .
\end{aligned}$$

The parameters entering the above DA's are borrowed from [18] whose values are $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$, and $\zeta_2^+ = 0$.

References

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Figure captions

Fig. (1) The dependence of the magnetic moment of the negative parity $\Sigma_0^b \rightarrow \Lambda_b^0$ transition on M^2 , at several fixed values of t , and at $s_0 = 40.0 \text{ GeV}^2$, in units of nuclear magneton μ_N .

Fig. (2) The same as Fig. (1), but at $s_0 = 42.5 \text{ GeV}^2$.

Fig. (3) The dependence of the magnetic moment of the negative parity $\Sigma_0^b \rightarrow \Lambda_b^0$ transition on $\cos \theta$, at several fixed values of M^2 , and at $s_0 = 40.0 \text{ GeV}^2$, in units of nuclear magneton μ_N .

Fig. (4) The same as Fig. (3), but at $s_0 = 42.5 \text{ GeV}^2$.

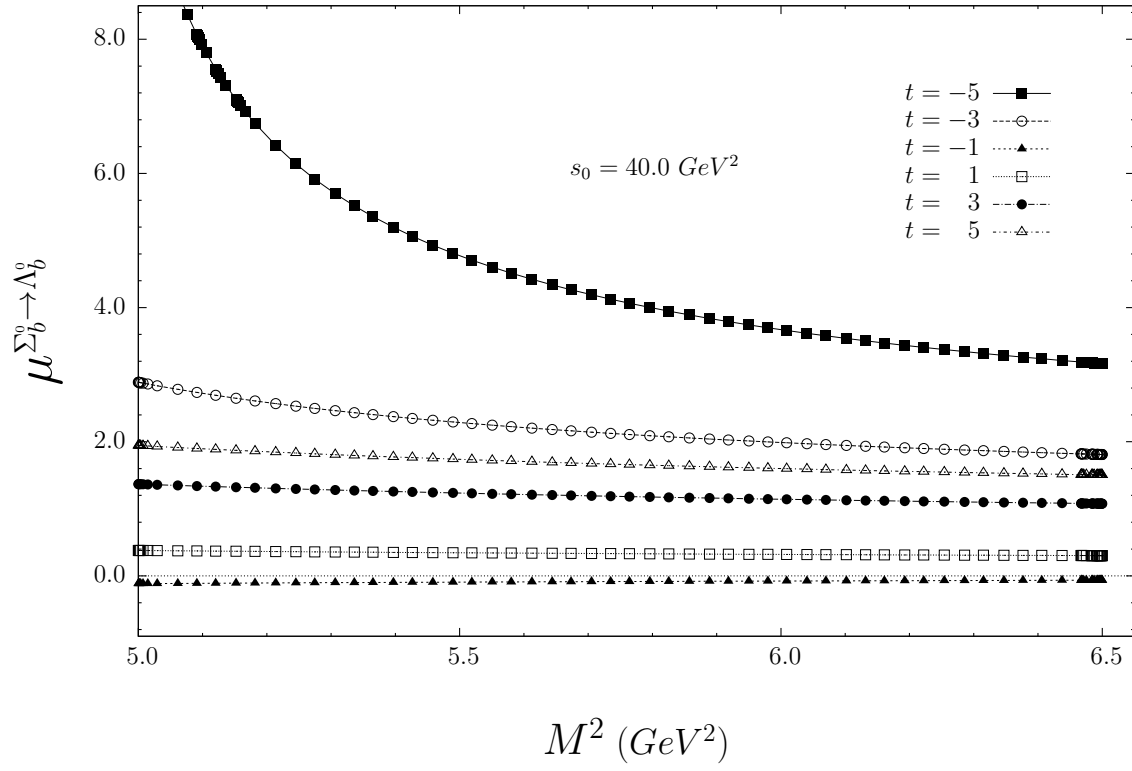


Figure 1:

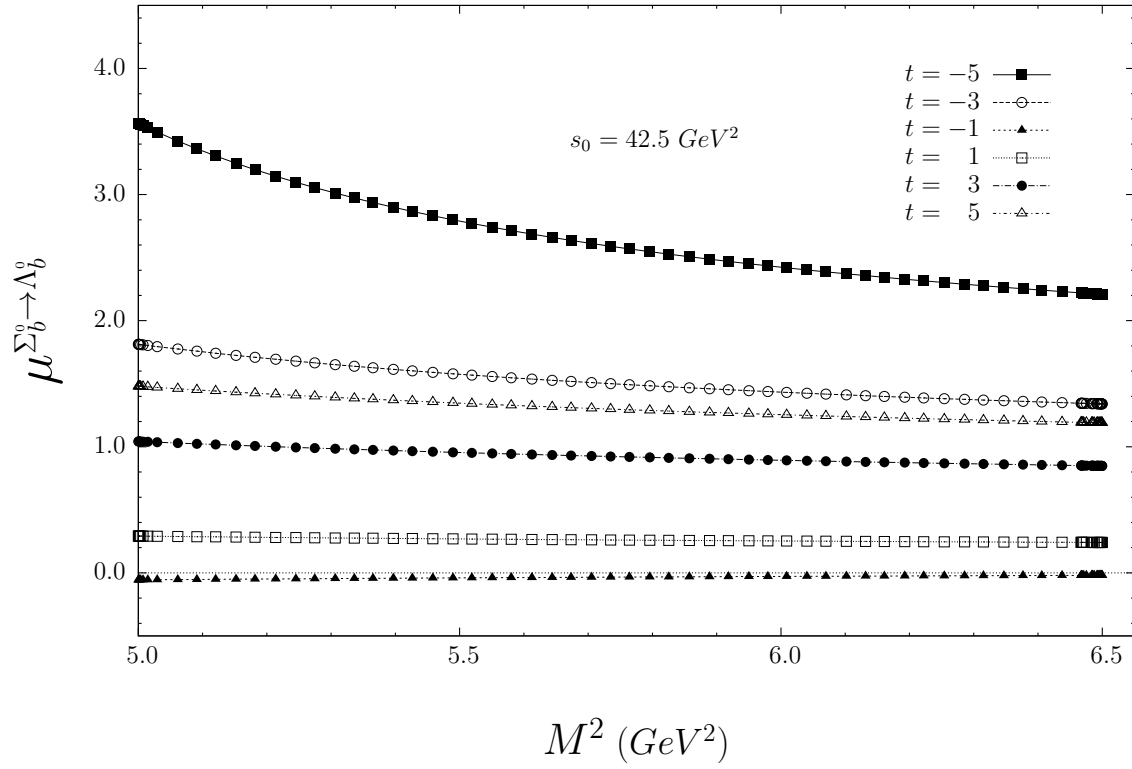


Figure 2:

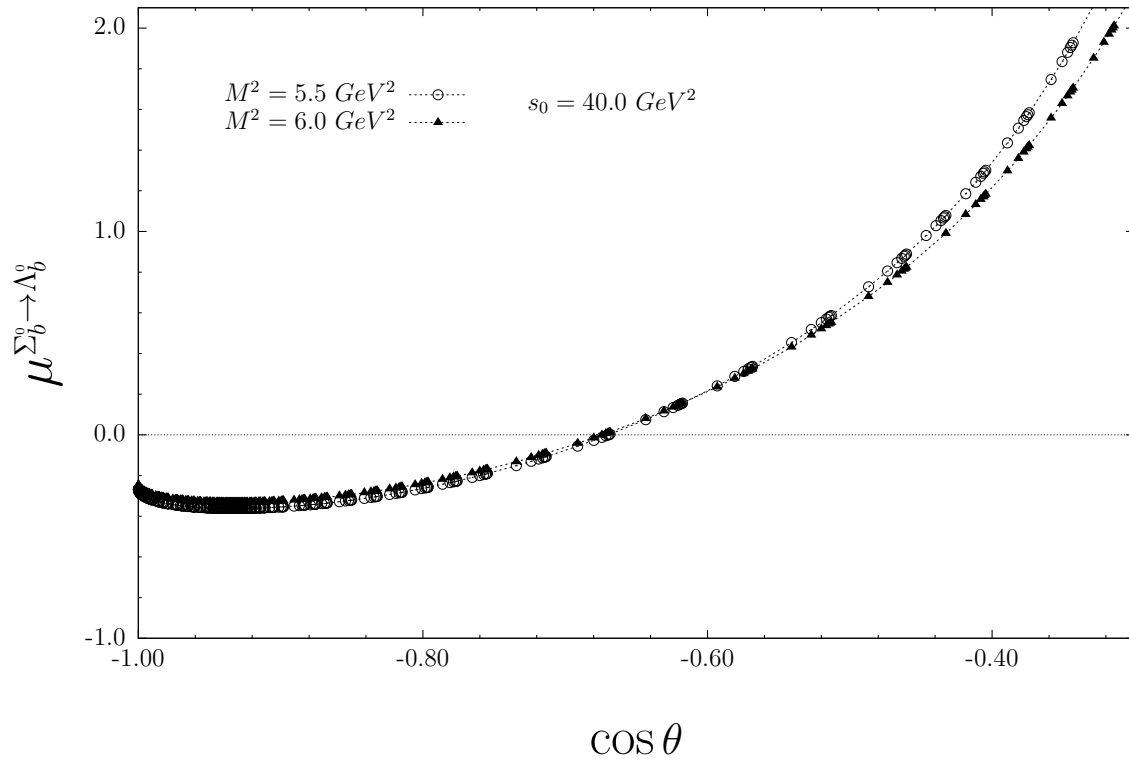


Figure 3:

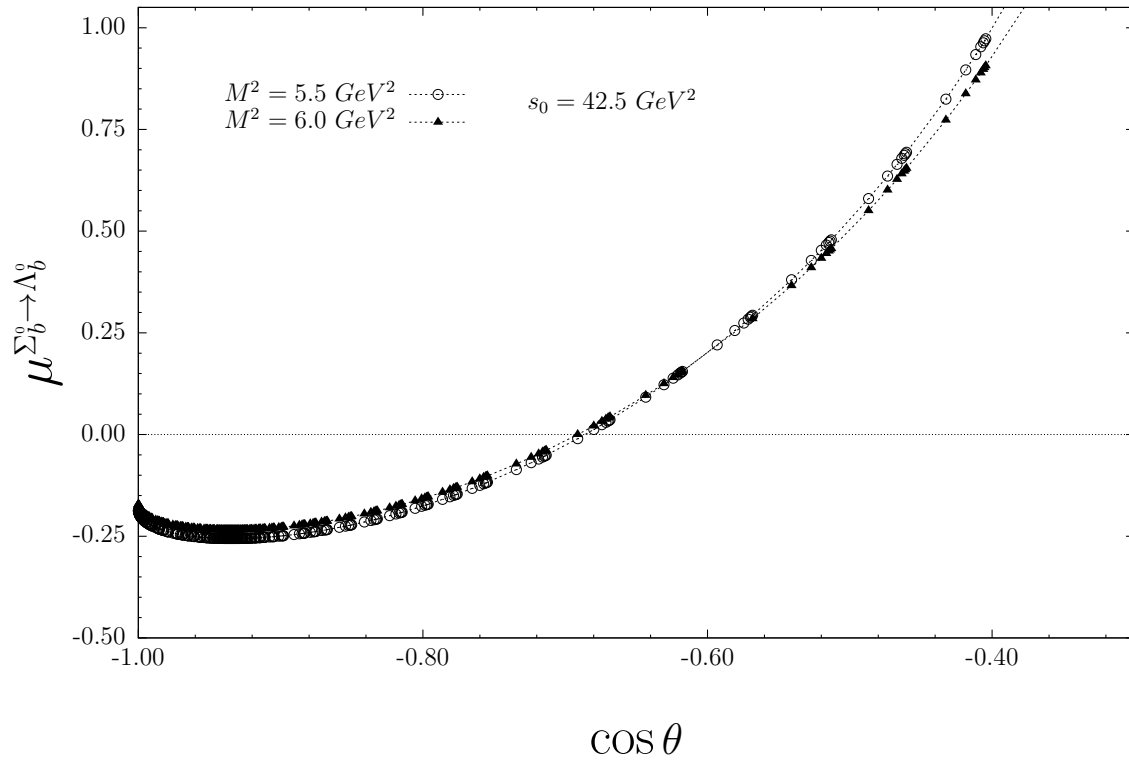


Figure 4: